Instrumental variables and two-stage least squares

Wooldridge ch. 15: when the regressor is not exogenous

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Today's goals

By the end of today you can:

1. Explain why OLS fails when the regressor is endogenous, and the two conditions an instrument must meet
2. Run IV and 2SLS in Stata, first stage first, and read the result
3. Judge an instrument: is it strong enough, is the exclusion believable, and what does IV estimate?

From last time

Answer on your own first, then compare with a neighbour

1. In Lab 1, did clustering make the AI coefficient unbiased?

The problem: the regressor is related to the error

- **Omitted variables:** ability raises both schooling and wages; good management raises both AI adoption and performance
- **Measurement error** in the regressor
- **Simultaneity:** y also affects x (feedback, as with the Riksbank)

Then OLS is biased and inconsistent, and more data does not help. We need variation in x that is unrelated to u.

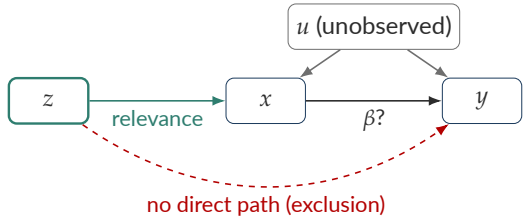
In symbols

Endogenous: $\text{Cov}(x, u) \neq 0 \Rightarrow \text{plim}\, \hat{\beta}_{OLS} = \beta + \text{Cov}(x, u)/\text{Var}(x) \neq \beta$

Key terms for today

- **Endogenous regressor:** related to the error u; **exogenous:** unrelated
- **Instrument** z**:** moves x (**relevance**) and affects y only through x (**exclusion**)
- **First stage:** x on z. **Reduced form:** y on z. **2SLS:** the two combined
- **Complier:** someone whose x the instrument changes

An instrument: variation in x unrelated to u



- **Relevance:** z moves x (testable: the first stage)
- **Exclusion:** z is unrelated to u, affects y only via x (not testable)

In symbols: key

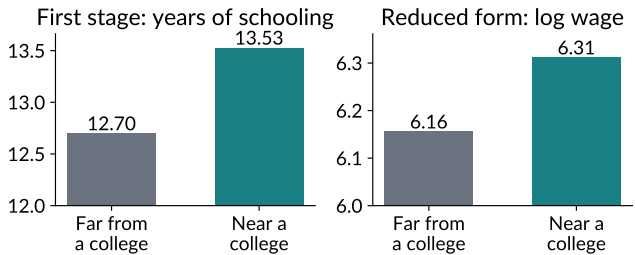
$$\text{Cov}(z,x) \neq 0, \quad \text{Cov}(z,u) = 0 \quad \Rightarrow \quad \hat{\beta}_{IV} = \widehat{\text{Cov}}(z,y) / \widehat{\text{Cov}}(z,x)$$

Textbook example: what is a year of school worth?

- Card (1995): 3,010 US men in 1976; y = log wage, x = years of schooling
- **The worry:** ability raises both schooling and wages, so OLS overstates the return
- **The instrument:** grew up near a four-year college (`nearc4`): college is cheaper to attend, so schooling rises, but proximity should not raise wages by itself
- OLS: 0.075 per year. Controls (`$controls`): experience and its square, race, city, region (as in Wooldridge)

Same data and coefficients as Wooldridge, Example 15.4 (our standard errors are robust).

First stage and reduced form, in two pictures



IV = right-hand gap / left-hand gap. With the controls the gaps are smaller (0.042 and 0.320): men near a college also lived in cities.

Step 1, first stage: does z move schooling?

```
regress educ nearc4 $controls, vce(robust)
```

```
      nearc4 |   .3198989   .0850763     3.76   0.000
```

- Near a college: 0.320 more years of schooling; first-stage F = 14.1

Always look at the first stage first. No first stage, no IV.

OLS gives 0.075 per year of school. IV gives...

Your bet first: hands up. The answer is on the next slide.

A. Less
B. About the same
C. More

Step 2, reduced form: does z move wages?

```
regress lwage nearc4 $controls, vce(robust)
```

```
      nearc4 |   .0420679   .0175211     2.40   0.016
```

- Near a college: 0.042 log points higher wages

In symbols: key

$x = \pi_0 + \pi_1 z + \ldots,\ y = \gamma_0 + \gamma_1 z + \ldots \ \Rightarrow\ \hat{\beta}_{IV} = \hat{\gamma}_1 / \hat{\pi}_1 = 0.042/0.320 = 0.132$

Step 3, 2SLS in one command

```
ivregress 2sls lwage (educ = nearc4) ///
    $controls, vce(robust)
```

```
        educ |   .1315038   .0539995     2.44   0.015
```

- IV: 0.132 against OLS 0.075; the standard error is about 15 times larger
- Larger, not smaller, than OLS: measurement error pulls OLS down, IV measures the effect for men the instrument moved, and IV is imprecise

2SLS = regress x on z and the controls, then y on the predicted x: done in one command so that the standard errors are right.

Weak instruments magnify any flaw

- With a weak instrument, IV is imprecise and biased towards OLS, and a small violation of exclusion becomes a large bias
- **Old rule:** first-stage F above 10. Here F = 14.1: passes
- **Newer result** (Lee, McCrary, Moreira and Porter 2022): the usual 1.96 is valid only when F reaches about 104.7. At F = 14.1 the critical value is about 2.94: Card's t = 2.44 falls short
- **A test that stays valid:** the reduced form's test of z (p = 0.016) still rejects "no effect"

A first stage that passes F = 10 can still be too weak for the usual t-test.

What does IV estimate? The compliers' effect



- The instrument moves schooling only for some: men who went further *because* a college was near (the "compliers"). Assumed: proximity never *lowers* anyone's schooling
- With effects that differ across people, IV estimates the effect for them: a local average treatment effect (Imbens and Angrist 1994)

Always ask: whose behaviour does my instrument change?

What we can test, and what we cannot

- **Relevance:** yes, the first stage (`estat firststage`)
- **Is x endogenous at all?** `estat endogenous` (OLS against IV; not the FE-against-RE Hausman of Lab 2): p = 0.27; the data cannot tell OLS and IV apart
- **Exclusion:** not testable. With two or more instruments, an overidentification test checks only whether they agree

The exclusion restriction is an argument, not a test result. Make it in words, with economics.

An instrument for AI adoption? A referee's view

- **Broadband, a university nearby, AI education:** they raise AI adoption, but also productivity and wages directly: exclusion fails
- **Shift-share idea** (Hummels, Jørgensen, Munch and Xiang 2014, offshoring in Denmark): a firm's earlier trade partners times world supply shocks; the firm cannot cause them
- **In your report:** propose an instrument only with an argument for both conditions, or explain why none is credible; that is a strong answer too

Check your understanding

Answer on your own first, then compare with a neighbour

Is the AI exposure of a firm's occupations a valid instrument for AI adoption in a wage regression?

Summary

1. Endogeneity biases OLS; an instrument must move x (relevance) and be unrelated to u (exclusion)
2. First stage, reduced form, then 2SLS; IV = reduced form / first stage, with larger standard errors
3. Check strength (F, and 104.7 for the usual t-test), argue exclusion in words, and ask whose behaviour the instrument moves

Next: Lab 2, at 10:15: panel methods, tests, and IV on the course data.

Test yourself before next time

Without AI: this is the exam competence

1. Name the two conditions for a valid instrument. Which can you test?
2. The first stage is 0.5 and the reduced form 0.1. What is the IV estimate?
3. Why can a weak instrument be worse than no instrument?
4. Give an instrument for AI adoption, and the strongest argument against it

Appendix

Three things you may meet in Stata

- **R^2 after IV** can be negative and has no use for judging the model (15-1c)
- **Robust standard errors with 2SLS:** `vce(robust)` or `vce(cluster id)` as with OLS (15-6)
- **Panel IV:** `xtivreg y (x = z) controls, fe` combines fixed effects and an instrument (15-8)

Why a weak instrument magnifies any flaw

In symbols

$\text{plim}\,\hat{\beta}_{IV} = \beta + \frac{\text{Corr}(z,u)}{\text{Corr}(z,x)} \cdot \frac{\sigma_u}{\sigma_x}$: a small $\text{Corr}(z,x)$ blows up any $\text{Corr}(z,u)$ (Wooldridge 15-1b)

Reading guide: chapter 15

- **15-1 Motivation, inference, poor instruments:** taught
- **15-2 IV in multiple regression; 15-3 2SLS, weak instruments:** taught
- **15-4 Errors in variables:** read on your own
- **15-5 Testing endogeneity and overidentification:** taught briefly
- **15-6 Heteroskedasticity; 15-7 time series; 15-8 panel data:** read on your own

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Wooldridge, *Introductory Econometrics*, 8th edition (2025).